

Data Structures Def.

mainly study

- static - preprocessing D.B.

tradeoffs s/t

- space s .

- query $(a) \rightarrow [0,1] t$.

- dynamic - update (a)

tradeoffs t/q

- query $(a) \rightarrow [0,1] t_q$

- space s

Dynamic: partial sums

eg list indexing

$A[1..n]$

histogram.

update $(i, x) \rightarrow A[i] = x$

query $(i) \rightarrow \sum_{j=1}^i A[j]$.

dynamic graph ~~problems~~ - connectivity

update - edges insert/delete edges.

query (u, v) u & v connected?

(un)

static tree. Update: mark node

query: is a certain node?

(ip-looking)

Models

Chromogrom [Freeman & saks '89]

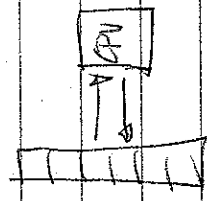
RAM w bits $w = \log n$

$\Omega(\log n / \log \log n)$ bands.



restricted version of partial sum $\{0,1\}^n$ array.

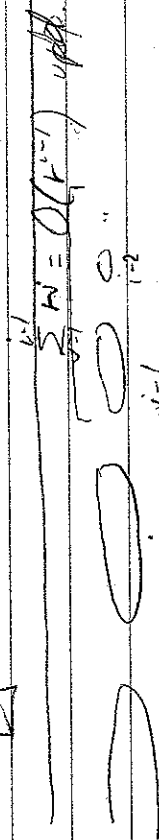
$\{+, \cdot\} \ll$ all operations in C
 sparse model for lower bounds.
 count each access to memory "cell probe".



n \times \times \times \times \times query (random i)

update (random i , random $\{0,1\}^w$)
 time $t_q = \Omega\left(\frac{\log n}{\log(w \log n)}\right)$

i t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10}
 epochs. $\log n$ epochs.
 groups upd. into epochs



$\rightarrow O(r^{i-1} t_0)$ cell writes.
 r^i upd. $O(r^{i-1} t_w)$ bits written
 r^i random bits.
 $r^{i-1} t_w \ll r^i$

Thm. $\forall i$ query probes $\Omega(i)$ cells from epochs i in exp. for some i

cell is from epoch i if most recent update during
 cell is from epoch i

Alg for the encoding problem

bits

$P = P_i$ random query reads cell from epoch i

Goal: $P = \Omega(i)$

For code - all cells written in epochs $< i$ ($r^i \cdot t_w$)

↳ $\text{dur.} \cdot \text{count} \cdot O(w)$

Problem: Consider r^i random queries

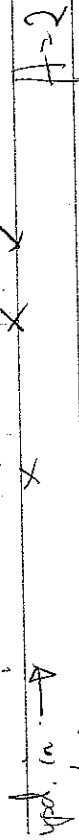
represent answer to those queries

- which queries need to read cell from epoch i

Claim: $\#(\text{answers}) = \Omega(r^i)$ bits $\#(\text{answers updates in all ep. except } i) = \Omega(r^i)$

$E[k] \cdot p \cdot r^i \cdot E[\text{size}] \log(p \cdot r^i) = r^i \cdot O(p \log \frac{1}{p})$

$A[1] \quad A[2] \quad A[n]$



$\# = 2$

answers to those queries k bits $E[\text{size}] = p \cdot r^i$

$E[\text{total size}] = O(r^i \cdot t_w \cdot w + r^i \cdot p \cdot \log \frac{1}{p}) = \Omega(r^i)$

Def: interleaves = # switches $v \rightarrow q$ in sorted list of q indices

Claim: $\# \geq$ interleaves

$\#(v_1, \dots, v_n) = \sum H(v_i, v_1, \dots, v_{i-1}) = \sum 1$

Decoding

- simulate queries not reading cell from epoch i

cell probe - epoch $< i$ to cell is in the encoding.
epoch $> i$ - cell contents is free

$E[\text{interleaves}] = \Omega(r^i)$